Closing Tues: HW 12.4, 13.2

Exam 2 is Thursday!

13.2 Definite Integrals (Continued)

Recall from last lecture:

Fundamental Theorem of Calculus

If $F(x)$ is *any* anti-derivative of $f(x)$, then

$$\int_a^b f(t)dt = F(b) - F(a)$$

Step 1: Find *any* antiderivative, $F(x)$.

Step 2: Evaluate $F(x)$ at $x = b$ and $x = a$.

Step 3: Subtract

$$\int_a^b f(x)dx = \quad F(x)\Big|_a^b = F(b) - F(a)$$

Entry Task: Evaluate

1. $\displaystyle\int_1^5 \frac{3}{4x^2}\, dx$

2. $\displaystyle\int_0^1 e^{x/3}\, dx$

$\boxed{1}$

$$\int_1^5 \frac{3}{4} x^{-2} dx$$

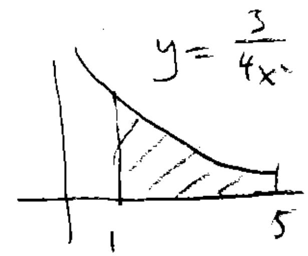
$$= \frac{3}{4} \frac{1}{-1} x^{-1} \Big|_1^5$$

$$= -\frac{3}{4} \frac{1}{x} \Big|_1^5$$

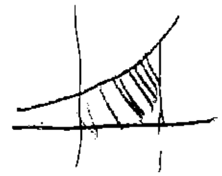
$$= -\frac{3}{4}\left(\frac{1}{5} - \frac{1}{1}\right)$$

$$= -3/4\left(\frac{1}{5} - \frac{5}{5}\right)$$

$$= -3/4\left(-4/5\right) = \boxed{3/5 = 0.6}$$



$\boxed{2}$ $\int_0^1 e^{\frac{1}{3}x}\,dx$

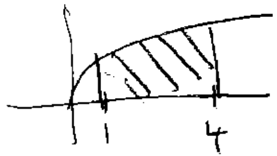


$= \frac{1}{1/3} e^{1/3 x} \Big|_0^1$

$= 3\left(e^{1/3} - e^0\right)$

$= 3\left(e^{1/3} - 1\right) \approx 1.1868$

3. $\int_1^4 \sqrt{x}\,dx$



$= \int_1^4 x^{1/2}\,dx$

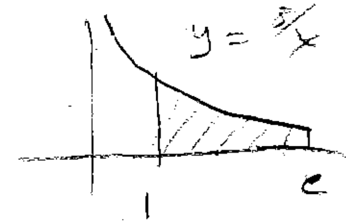
$= \frac{2}{3} x^{3/2} \Big|_1^4$

$= \frac{2}{3}\left(4^{3/2} - 1^{3/2}\right)$

$= \frac{2}{3}(8-1)$

$= \boxed{14/3}$

4. $\int_1^e \frac{5}{x}\,dx$



$= \int_1^e 5 \cdot \frac{1}{x}\,dx$

$= 5\ln(x)\Big|_1^e$

$= 5(\underbrace{\ln(e)}_{1} - \underbrace{\ln(1)}_{0})$

$= \boxed{5}$

EXAM 1 IS THURSDAY IN QUIZ SECTION

Allowed:

1. A **Ti-30x IIS Calculator**
2. An 8.5 by 11 inch sheet of handwritten notes (front/back)
3. A pencil or black/blue pen (and a ruler)

Details and rules:

1. 4 pages of questions, 50 minutes, use your time effectively.

2. **Show your work using methods from class.** The correct answer with no supporting work is worth zero points.

3. Clearly indicate work you want graded. Put a box around your final answers.

4. No make-up exams; if you are physically unable to be at the test, go to doctor and get documentation (and your grade will be prorated)

5. There are multiple versions of the test!!!! They will look similar. If you copy off of a classmate we will know and we will report to the student misconduct board (and you'll get a zero on the entire test). So don't sit next to your study partners and don't be tempted to copy off a classmate.

Quick Review (Checklist)

11.1/11.2: New Derivative Skills

We added

$$\frac{d}{dx}\left(e^{f(x)}\right) = e^{f(x)} f'(x)$$

$$\frac{d}{dx}(\ln(x)) = \frac{1}{f(x)} f'(x)$$

Be able to use these in combination with our other rules. Two examples from homework:

1. $y = (e^{4x} + 5)^{10}$
2. $y = x^3 \ln\left(1 + \sqrt{x}\right)$

$\downarrow$

$x^{1/2}$

$\boxed{1}$ $\boxed{y' = 10(e^{4x}+5)^9 \cdot e^{4x} \cdot 4}$

$= 40e^{4x}(e^{4x}+5)^9$

$\boxed{2}$

$\boxed{y' = x^3 \frac{1}{1+\sqrt{x}} \frac{1}{2}x^{-1/2} + 3x^2 \ln(1+\sqrt{x})}$

$= \frac{x^3}{2(1+\sqrt{x})\sqrt{x}} + 3x^2\ln(1+\sqrt{x})$

12.1/12.3, 13.2: Anti-derivative Skills

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$$

$$\int \frac{1}{x}\,dx = \ln(x) + C$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$$

Step 1: Expand and Simplify

Step 2: Use the rules above (don't forget +C)

Step 3: Check your answer (derivative)

Step 4: If it is a definite integral, evaluate and subtract.

Three examples:

1. $\int \frac{5}{x} - 3e^{4x}\,dx = \boxed{5\ln(x) - \frac{3}{4}e^{4x} + C}$

2. $\int \frac{x+2}{x^6}\,dx = \int x^{-5} + 2x^{-6} = \boxed{\frac{1}{-4}x^{-4} - \frac{2}{5}x^{-5} + C}$

$= -\frac{1}{4x^4} - \frac{2}{5x^5} + C$

3. $\int_0^4 5 + \sqrt{x}\,dx$

$= \int_0^4 5 + x^{1/2}\,dx$

$= 5x + \frac{2}{3}x^{3/2}\Big|_0^4$

$= \left(5(4) + \frac{2}{3}(4)^{3/2}\right) - \left(5(0) + \frac{2}{3}(0)^{3/2}\right)$

$= 20 + \frac{16}{3}$

$= \frac{60}{3} + \frac{16}{3} = \boxed{\frac{76}{3}}$

10.1-10.3, 12.4: Analyzing Functions

First:
What are you given and what do you want? What is the `original' function? You may need to use derivative/anti-derivative skills to find the function you want!

Second: Translate

Problem Type 1:
To find *critical numbers, horizontal tangents, local max/min*, or *increasing/decreasing*

1. Solve $f'(x) = 0$
2. Draw 1st Derivative number line (figure out when 1st derivative is positive or negative)
3. Make appropriate conclusions.

(*Note*: To determine local max/min, you can also use the 2nd deriv. test as a short-cut).

Problem Type 2:
To find *points of inflection, concave up/down.*

1. Solve $f''(x) = 0$
2. Draw 2nd Derivative number line (figure out when 2nd derivative is positive or negative)

Problem Type 3:
To find *global max/min* on a given interval

1. Solve $f'(x) = 0$
2. Plug critical numbers and endpoints into the original function.

Third: Interpret and present your answer. Reread the question. Did you answer it and give the answer in the desired form?

10.3, 12.4: Special Applications

- Know when and how to do derivatives and antiderivatives in applications:
  1. TR/MR, TC/VC/MC, P/MP,
  2. amount of water in a vat / rate of flow
  3. height / rate of ascent,
  4. dist / speed

- For antiderivatives, know how to use initial conditions to find the constant of integration C.

- Know how to look at a graph of a derivative to make conclusions about antiderivatives. Be able to find and interpret the *net area under a curve*.

- Know how to look at the graph of an "original" function and analyze slopes to make conclusions about the derivative.

Essential algebra skills

1. Rewriting powers, expanding, simplifying
2. Solving equations
   - clear the denominator
   - powers/roots, exponentials/logs
   - factoring
   - quadratic formula

Two Random Old Midterm Questions

1. Find *all* critical points for the function

$$f(x) = 5x + \frac{3}{x} + 3$$

and use the second derivative test to classify the critical points as local maxima or local minima. Clearly label your answers.

$$f(x) = 5x + 3x^{-1} + 3$$

$$f'(x) = 5 - 3x^{-2} = 5 - \frac{3}{x^2}$$

$$5 - \frac{3}{x^2} \stackrel{?}{=} 0, \text{ multiply by } x^2$$

$$\Rightarrow 5x^2 - 3 = 0$$

$$\Rightarrow 5x^2 = 3$$

$$x^2 = 3/5$$

$$x = \pm\sqrt{3/5}$$

$$f''(x) = 6x^{-3} = \frac{6}{x^3}$$

For $x = -\sqrt{3/5}$, $f''(-\sqrt{3/5}) = \frac{6}{(-\sqrt{3/5})^3}$ which is negative.

So $f'(-\sqrt{3/5}) = 0$ AND $f''(-\sqrt{3/5}) < 0$

HORIZ. TANGENT — CONCAVE DOWN

THUS $x = -\sqrt{3/5}$ gives a local max

For $x = \sqrt{3/5}$, $f''(\sqrt{3/5}) = \frac{6}{(\sqrt{3/5})^3}$ is positive!

So $f'(\sqrt{3/5}) = 0$ AND $f''(\sqrt{3/5}) > 0$

HORIZ. TANGENT — CONCAVE UP

THUS, $x = \sqrt{3/5}$ gives a local min.

2. Suppose $A'(t) = t^2 - 8t + 12$ is the rate of change in the amount of water in a vat, where *t* is in hours and $A'(t)$ is in gallons per hour. Assume the vat contains 100 gallons of water at time t=0.

(a) Find the formula, $A(t)$, for amount of water in the vat at time *t*.

(b) Find the maximum amount of water in the vat between *t = 0* and *t = 7* hours

(a) $A(t) = \int t^2 - 8t + 12 \, dt$

$A(t) = \frac{1}{3}t^3 - 4t^2 + 12t + c$

$A(0) = 100 \Rightarrow \frac{1}{3}(0)^3 - 4(0)^2 + 12(0) + c = 100$

$\Rightarrow c = 100$

$\boxed{A(t) = \frac{1}{3}t^3 - 4t^2 + 12t + 100}$

(b) $A'(t) = t^2 - 8t + 12 \stackrel{?}{=} 0$

$(x - 6)(x - 2) \stackrel{?}{=} 0$

$x = 2$ or $x = 6$

$A(0) = 100$

$A(2) = \frac{1}{3}(2)^3 - 4(2)^2 + 12(2) + 100 = 110.\overline{6}$

$A(6) = \frac{1}{3}(6)^3 - 4(6)^2 + 12(6) + 100 = 100$

$A(7) = 102.\overline{3}$

$\boxed{\text{MAXIMUM} = 110.\overline{6} \text{ gallons}}$